## 2007 REVISED

## SECONDARY MATHEMATICS

CORE CURRICULUM

#### Introduction

Students graduating from Utah high schools face a complex, technological, and constantly changing world. To compete successfully in the growing worldwide economy, students must have adequate preparation in the skills and understanding mathematics provides. Mathematics literacy is essential and the need for it is universal. The Utah State Secondary Mathematics Core outlines the essential skills and understanding required of capable adults.

The goal of the Core Curriculum is to develop mathematical proficiency in **every** student by building a conceptual base and developing mathematical fluency. Students who understand mathematics will be able to communicate their reasoning, use multiple representations, and think logically. They will develop positive attitudes toward mathematics, solve problems, and think creatively while connecting mathematics to other disciplines and to life. Students will use mathematical tools, such as manipulative materials and technology, to develop conceptual understanding and solve problems.

The Secondary Mathematics Core describes what students should know and be able to do at the end of each of the six core courses: Math 7, Pre-Algebra, Algebra 1, Geometry, Algebra 2, and Precalculus. Every standard and objective is essential and will be tested. This does not suggest that all objectives are of equal importance in developing students' proficiency, nor that they should receive an equal amount of time in the classroom.

The Secondary Mathematics Core was developed and revised by a community of Utah mathematics teachers, mathematicians, university mathematics educators, and State Office of Education specialists. It was critiqued by an advisory committee representing a wide variety of people from the community, as well as an external review committee. The Core reflects the current philosophy of mathematics education as expressed in national documents developed by the National Council of Teachers of Mathematics, the American Statistical Association, the College Board, and Achieve. This Mathematics Core has the endorsement of the Utah Council of Teachers of Mathematics. The Core reflects high standards of achievement in mathematics for all students.

#### **Key Components of Teaching and Learning Mathematics**

#### **Teachers**

Delivery of the core requires highly knowledgeable and qualified teachers in every secondary mathematics classroom. Mathematics teachers must be well prepared with an extensive knowledge of both mathematics and pedagogy. They must have an understanding of students and student learning and be able to adapt classroom instruction to meet student needs.

#### **Students**

Students in mathematics classrooms must take responsibility for their learning while receiving strong support from teachers, parents, and an informed society that recognizes the importance of a comprehensive mathematics education. They will understand mathematics more deeply through participation in activities that build and strengthen a profound understanding of mathematics and applications of mathematics. Their knowledge will be further enhanced through connections to prior learning and other disciplines.

#### Assessment

Assessment is an integral part of the curriculum and a routine part of classroom instruction. It must be rich and varied, consisting of both formative assessments that are used to inform instruction and summative assessments that are used to gauge student learning. Assessments provide students, teachers, and parents with important information about student progress and classroom effectiveness.

#### **Technology**

The purpose of technology is to enhance the investigation and modeling of a wide variety of mathematical concepts and engage students in the learning process. Technology must be integrated in the curriculum and used appropriately as part of mathematical instruction and assessment. Technology facilitates the organization and analysis of data, and efficiency and accuracy in computation and, used appropriately, has been shown to be a tool that can support the development of flexibility in the use of various representations. It is used to provide visual images leading to understanding of mathematical ideas and concepts.

#### **Course Articulation**

The Utah State Secondary Mathematics Core provides one course sequence for all students; however, the course in which students enter the sequence in the seventh grade may differ depending on individual student readiness. The sequence of the courses is Math 7, Pre-Algebra, Algebra 1, Geometry, Algebra 2, Precalculus, and AP Calculus and/or AP Statistics. Students who wish to complete AP Calculus or AP Statistics before graduation should be enrolled in Algebra 1 by eighth grade. Students who take four years of mathematics in high school and complete Precalculus will be well prepared to enter college doing college-level mathematics or to pursue other post-secondary experiences.

The initial placement of students in a seventh grade mathematics course has far-reaching implications. The most appropriate placement must take into consideration the student's level of cognitive development, emotional maturity, work ethic, and study habits. Success in algebra depends on a solid conceptual understanding of arithmetic and rational numbers, obtained through mastery of the Utah State Elementary Core.

The Utah State Office of Education has also defined several applied, advanced, and supplementary courses for students in need of either remediation or acceleration. Course titles and syllabi are available on the USOE web site.

#### **Organization**

The Core is designed to help teachers organize and deliver instruction.

- ❖ Each grade level begins with a course description emphasizing key concepts.
- ❖ The **Intended Learning Outcomes** (ILOs) describe the skills and attitudes students should acquire as a result of successful mathematics instruction. They are an integral part of the Core.
- ❖ A **Standard** is a broad statement of what students are expected to understand. Several Objectives are listed under each Standard.
- ❖ An **Objective** is a more focused description of what students need to know and be able to do at the completion of instruction. If students have mastered the Objectives associated with a given Standard, they have mastered that Standard for that course.
- ❖ Indicators are observable or measurable student actions that enable students to master an Objective. Indicators can help guide classroom instruction.

### Course Requirements

- 1. Students may complete a combination of core and applied, advanced, and supplemental (AAS) courses to meet the minimum graduation requirements. A minimum of two credits must be earned from the core sequence. AAS course listings are available on the Secondary Mathematics web page.
- 2. No student may obtain two high school mathematics credits (9-12) for completing the same course.
- 3. Students may not take a course for mathematics graduation credit that is a prerequisite of a previously completed secondary mathematics course (7-12). The prerequisite of each course is listed at the beginning of each course description.
- 4. Courses at algebra level or above may be used for graduation credit.
- 5. Students should receive appropriate counseling as they register for mathematics courses so that they will be able to complete the current graduation requirements for mathematics, and to make sure they will have the mathematical training needed to succeed in the post-secondary training of their choice.
- 6. Finishing a mathematics course beyond Algebra 2 is a key predictor of collegiate success and completion (U.S. Department of Education, *The Toolbox Revisited: Paths to Degree Completion from High School Through College*, 2006).

### **Course Offerings Available by Grade**

### 7<sup>th</sup> Grade

Math 7

Pre-Algebra

Algebra 1

### 8<sup>th</sup> Grade

Pre-Algebra

Algebra 1

Geometry

### 9<sup>th</sup> Grade

Algebra A (AAS)

Algebra 1

Geometry

Algebra 2

### 10<sup>th</sup> Grade

Algebra B

Algebra 1

Geometry

Algebra 2

Precalculus

**AAS Courses** 

### 11<sup>th</sup> Grade

Algebra 1

Geometry

Algebra 2

Precalculus

AP Calculus

**AP Statistics** 

AAS courses

### 12<sup>th</sup> Grade

Algebra 1

Geometry

Algebra 2

Precalculus

**AP Calculus** 

**AP Statistics** 

AAS courses

#### **Intended Learning Outcomes**

The main intent of mathematics instruction at the secondary level is for students to develop mathematical proficiency that will enable them to efficiently use mathematics to make sense of and improve the world around them.

The Intended Learning Outcomes (ILOs) describe the skills and attitudes students should acquire as a result of successful mathematics instruction. They are an essential part of the Mathematics Core Curriculum and provide teachers with a standard for student learning in mathematics.

The ILOs for mathematics at the secondary level are:

- 1. Develop positive attitudes toward mathematics, including the confidence, creativity, enjoyment, and perseverance that come from achievement.
- 2. Become proficient problem-solvers by posing appropriate questions, selecting appropriate methods, employing a variety of strategies, and exploring alternative approaches.
- 3. Think logically, using inductive reasoning to formulate reasonable conjectures and using deductive reasoning for justification, formally and informally.
- 4. Cooperatively and independently explore mathematics, using inquiry and technological skills.
- 5. Make connections between mathematical ideas, between mathematics and other disciplines, and to life.
- 6. Communicate mathematics through verbal, written, and visual representations, using precise mathematical language and symbolic notation.

## Math 7 Prerequisite: Sixth Grade Mathematics

Students in Math 7 will study mathematics concepts from sixth grade in more depth and extend knowledge to basic pre-algebra by conjecturing, verifying, thinking critically, and applying mathematical concepts. This course focuses on computation and estimation with rational numbers and emphasizes proportional reasoning. Students will investigate and explore mathematical ideas using technology and models to develop multiple strategies for analyzing complex situations. Students will apply mathematical skills and make meaningful connections to life's experiences.

Standard I: Students will expand number sense to understand, perform operations, and solve problems with rational numbers.

#### Objective 1: Represent rational numbers in a variety of ways.

- a. Demonstrate multiple ways to represent whole numbers, decimals, fractions, percents, and integers using models and real-life examples.
- b. Simplify numerical expressions with whole number exponents using order of operations, and recognize that any positive number to the 0 power is 1.
- c. Represent numbers greater than one using scientific notation.
- d. Select the most appropriate form of a rational number for a given context.

### Objective 2: Compare and order rational numbers, including positive and negative fractions, positive and negative mixed numbers, and positive and negative decimals.

- a. Identify, read, and locate rational numbers on a number line.
- b. Compare pairs of rational numbers in different forms.
- c. Order rational numbers with and without a number line.

#### Objective 3: Explain relationships and equivalences among rational numbers.

- a. Find equivalent forms for common fractions, decimals, percents, and ratios, including repeating or terminating decimals.
- b. Predict the effect of operating with fractions, decimals, percents, and integers as an increase or a decrease of the original value.
- c. Recognize and use the identity properties of addition and multiplication, the multiplicative property of zero, the commutative and associative properties of addition and multiplication, and the distributive property of multiplication over addition.
- d. Recognize and use the inverse operations of adding and subtracting a fixed number, multiplying and dividing by a fixed number, and computing squares of whole numbers and taking square roots of perfect squares.

### Objective 4: Model meanings of ratios and operations with rational numbers.

- a. Demonstrate that the fraction  $\frac{a}{b}$  represents a divided by b.
- b. Recognize percents as ratios based on 100 and decimals as ratios based on powers of 10.
- c. Extend the multiplication of whole numbers to multiplication of fractions using area models, measurement models, and the number line.
- d. Compare the division of whole numbers to the division of fractions using area or set models, the number line, and missing factors.

#### **Objective 5: Solve problems involving rational numbers.**

- a. Compute fluently using all four operations with integers and positive fractions and decimals.
- b. Solve problems using factors, multiples, prime factorization, relatively prime numbers, and common divisibility rules.
- c. Solve application problems involving rational numbers.
- d. Determine if an answer is reasonable using estimation.

### Mathematical Language and Symbols Students Should Use

whole number, decimal, fraction, percent, integer, exponent, scientific notation, rational number, identity, commutative, associative, distributive, factor, multiple, prime, relatively prime, additive inverse, multiplicative inverse

### Standard II: Students will use proportional reasoning to solve problems.

### Objective 1: Solve problems involving ratios, rates, proportions and percentages.

- a. Solve ratio and rate problems using informal methods involving multiplication and division.
- b. Solve percent problems using ratio and proportion, including problems involving discounts, interest, taxes, tips, and percent increase or decrease.
- c. Solve problems involving proportions, rates, and measures.

### Objective 2: Apply the properties of proportionality to different units of measurement.

- a. Convert from one unit of measurement to an equivalent unit of measurement in the same system using a given conversion factor.
- b. Understand that in a proportional relationship, all dimensions change by the same scale factor.
- c. Create and interpret scale drawings and approximate distance on maps using proportions.

Mathematical Language and Symbols Students Should Use

ratio, rate, proportion, scale drawing, conversion factor

### Standard III: Students will develop fluency with the language and operations of algebra to analyze and represent relationships.

#### Objective 1: Evaluate, simplify, and solve algebraic expressions and equations.

- a. Write a variable expression to identify pattern relationships, and use those expressions to make predictions.
- b. Translate verbal expressions into algebraic expressions.
- c. Simplify and evaluate algebraic expressions.
- d. Show that performing the same operation on both sides of an equation will produce an equivalent equation.
- e. Solve single-variable linear equations and inequalities of the form ax + b = c, ax + b < c, or ax + b > c.

#### Objective 2: Represent relationships using graphs, tables, and other models.

- a. Identify integer coordinates when given the graph of a point on a rectangular coordinate system.
- b. Graph ordered pairs of integers on a rectangular coordinate system.
- c. Model real-world problems using graphs, tables, equations, manipulatives, and pictures.

#### Mathematical Language and Symbols Students Should Use

variable expression, algebraic expression, equivalent, linear equation, linear inequality, rectangular coordinate system, ordered pair

### Standard IV: Students will use algebraic, spatial, and logical reasoning to solve geometry and measurement problems.

### Objective 1: Draw, label, and describe attributes of geometric figures to determine geometric relationships.

- a. Draw, label, and describe relationships among line segments, rays, lines, parallel lines, and perpendicular lines, including midpoint of a line segment.
- b. Draw, label, and describe relationships among vertical, adjacent, complementary, and supplementary angles.
- c. Draw, label, and describe attributes of angles, triangles, and quadrilaterals.

## Objective 2: Determine measurements in metric and customary units using appropriate tools and formulas.

- a. Estimate metric and customary measures using everyday objects and comparisons.
- b. Measure length, area, volume, and angles to appropriate levels of precision.
- c. Calculate the measurement of everyday objects using formulas for perimeters and areas of triangles and quadrilaterals, and circumferences and areas of circles.
- d. Calculate the measurement of everyday objects using formulas for surface area and volume of right triangular and rectangular prisms and cylinders.

#### Mathematical Language and Symbols Students Should Use

line segment, ray, line, parallel, perpendicular, midpoint, vertical angles, adjacent angles, complementary angles, supplementary angles

### Standard V: Students will understand concepts from probability and statistics and apply statistical methods to solve problems.

### Objective 1: Use basic concepts of probability to determine the likelihood of an event and compare the results of various experiments.

- a. Write the results of a probability experiment as a fraction, ratio, or decimal, between zero and one, or as a percent between zero and one hundred, inclusive.
- b. Compare experimental results with theoretical probability.
- c. Compare individual, small group, and large group results of a probability experiment.

### Objective 2: Display and compare data to make predictions and formulate conclusions.

- a. Display data using tables, scatter plots, and circle graphs.
- b. Compare two similar sets of data on the same graph.
- c. Compare two different kinds of graphs representing the same set of data.
- d. Propose and justify inferences and predictions based on data.

Mathematical Language and Symbols Students Should Use experimental result, theoretical probability, scatter plot, circle graph, inference

### Pre-Algebra

#### Prerequisite: Proficiency in Sixth Grade Mathematics or Math 7

The goal of Pre-Algebra is to develop fluency with rational numbers and proportional relationships. Students will extend their elementary skills and begin to learn algebra concepts that serve as a transition into formal Algebra and Geometry. Students will learn to think flexibly about relationships among fractions, decimals, and percents. Students will learn to recognize and generate equivalent expressions and solve single-variable equations and inequalities. Students will investigate and explore mathematical ideas using technology and models to develop multiple strategies for analyzing complex situations. Students will analyze situations verbally, numerically, graphically, and symbolically. Students will apply mathematical skills and make meaningful connections to life's experiences.

### Standard I: Students will expand number sense to understand, perform operations, and solve problems with rational numbers.

### Objective 1: Compute fluently with understanding and make reasonable estimates with rational numbers.

- a. Compute fluently using all four operations with integers, and explain why the corresponding algorithms work.
- b. Compute fluently using all four operations with rational numbers, including negative fractions and decimals, and explain why the corresponding algorithms work.
- c. Check the reasonableness of results using estimation.

### Objective 2: Analyze relationships among rational numbers, including negative rational numbers, and operations involving these numbers.

- a. Order rational numbers in various forms, including scientific notation (positive and negative exponents), and place numbers on a number line.
- b. Predict the effect of operating with fractions, decimals, percents, and integers as an increase or a decrease of the original value.
- c. Recognize and use the identity properties of addition and multiplication, the multiplicative property of zero, the commutative and associative properties of addition and multiplication, and the distributive property of multiplication over addition.
- d. Recognize and use the inverse operations of adding and subtracting a fixed number, multiplying and dividing by a fixed number, and computing squares of whole numbers and taking square roots of perfect squares.

### Objective 3: Solve problems involving rational numbers using addition, subtraction, multiplication, and division.

- a. Recognize the absolute value of a rational number as its distance from zero.
- b. Simplify numerical expressions, including those with whole number exponents and absolute values, using the order of operations.
- c. Solve problems involving rational numbers, percents, and proportions.

#### **Mathematical Language and Symbols Students Should Use**

integer, rational, scientific notation, identity, commutative, associative, distributive, square, square root, absolute value, order of operations,  $a^b$ 

#### Standard II: Students will use proportion and similarity to solve problems.

#### Objective 1: Model and illustrate meanings of ratios, percents, and decimals.

- a. Compare ratios to determine if they are equivalent.
- b. Compare ratios using the unit rate.
- c. Represent percents as ratios based on 100 and decimals as ratios based on powers of ten
- d. Graph proportional relationships and identify the unit rate as the slope of the related line.

### Objective 2: Solve a wide variety of problems using ratios and proportional reasoning.

- a. Set up and solve problems involving proportional reasoning using variables.
- b. Solve percent problems, including problems involving discounts, interest, taxes, tips, and percent increase or decrease.
- c. Solve ratio and rate problems using informal methods.

### Objective 3: Recognize similar polygons and use properties of similar triangles to solve problems and define the slope of a line.

- a. Define similar polygons as polygons with corresponding angles congruent and corresponding sides that are proportional.
- b. Identify pairs of similar triangles using two pairs of congruent angles, or two pairs of proportional sides with congruent included angles.
- c. Find missing lengths of similar triangles, including inaccessible lengths, using proportions.
- d. Define the slope of a line as the ratio of the vertical change to the horizontal change between two points, and show that the slope is constant using similarity of right triangles.

### Mathematical Language and Symbols Students Should Use ratio, proportion, variable, similar polygon, similar triangle, congruent, slope

### Standard III: Students will develop fluency with the language and operations of algebra to analyze and represent relationships.

#### Objective 1: Generalize and express patterns using algebraic expressions.

- a. Compare representations of a relation using tables, graphs, algebraic symbols, and mathematical rules.
- b. Describe simple patterns using a mathematical rule or algebraic expression.
- c. Create and extend simple numerical and visual patterns.

### Objective 2: Evaluate, simplify, and solve algebraic expressions, equations, and inequalities.

- a. Evaluate algebraic expressions, including those with whole number exponents, when given values for the variable(s).
- b. Simplify algebraic expressions using the order of operations, algebraic properties, and exponent rules.
- c. Solve single-variable linear equations and inequalities, including those that must be simplified on one side or those with variables on both sides of an equation.

#### Objective 3: Represent relationships using graphs, tables, and other models.

- a. Identify approximate rational coordinates when given the graph of a point on a rectangular coordinate system.
- b. Graph ordered pairs of rational numbers on a rectangular coordinate system.
- c. Graph linear equations using ordered pairs or tables.
- d. Recognize that all first order equations produce linear graphs.
- e. Model real-world problems using graphs, tables, equations, manipulatives, and pictures, and identify extraneous information.

## Mathematical Language and Symbols Students Should Use table, algebraic expression, equation, linear, ordered pair, extraneous information

## Standard IV: Students will use algebraic, spatial, and logical reasoning to solve geometry and measurement problems.

#### Objective 1: Apply the properties of proportionality of different units of measure.

- a. Convert units of measure within the same system.
- b. Create and interpret scale drawings and approximate distance on maps using scale factors.
- c. Solve problems using scale factors.

### Objective 2: Derive formulas for surface areas and volume of three-dimensional figures.

- a. Derive formulas for and calculate surface area and volume of right prisms and cylinders using appropriate units.
- b. Explain that if a scale factor describes how corresponding lengths in two similar objects are related, then the square of the scale factor describes how corresponding areas are related and the cube of the scale factor describes how corresponding volumes are related.
- c. Find lengths, areas, and volumes of similar figures, using the scale factor.
- d. Select appropriate two- and three-dimensional figures to model real-world objects, and solve a variety of problems involving surface areas and volumes of cylinders and prisms.

### Mathematical Language and Symbols Students Should Use unit of measure, scale, scale factor, surface area, volume, prism, cylinder

### Standard V: Students will understand concepts from probability and statistics and apply statistical methods to solve problems.

### Objective 1: Calculate probabilities of events and compare theoretical and experimental probability.

- a. Solve counting problems using the Fundamental Counting Principle.
- b. Calculate the probability of an event or sequence of events with and without replacement using models.
- c. Recognize that the sum of the probability of an event and the probability of its complement is equal to one.
- d. Make approximate predictions using theoretical probability and proportions.
- e. Collect and interpret data to show that as the number of trials increases, experimental probability approaches the theoretical probability.

### Objective 2: Formulate questions and answer the questions by organizing and analyzing data.

- a. Formulate questions that can be answered through data collection and analysis.
- b. Determine the 25<sup>th</sup> and 75<sup>th</sup> percentiles (first and third quartiles) to obtain information about the spread of data.
- c. Graphically summarize data of a single variable using histograms and box-and-whisker plots.
- d. Compute the mean and median of a numerical characteristic and relate these values to the histogram of the data.
- e. Use graphical representations and numerical summaries to answer questions and interpret data.

#### Mathematical Language and Symbols Students Should Use

Fundamental Counting Principle, complement, theoretical probability, experiment, data, percentile, histogram, box-and-whisker plot, spread

## Algebra 1 Prerequisite: Pre-Algebra

The main goal of Algebra is to develop fluency in working with linear equations. Students will extend their experiences with tables, graphs, and equations and solve linear equations and inequalities and systems of linear equations and inequalities. Students will extend their knowledge of the number system to include irrational numbers. Students will generate equivalent expressions and use formulas. Students will simplify polynomials and begin to study quadratic relationships. Students will use technology and models to investigate and explore mathematical ideas and relationships and develop multiple strategies for analyzing complex situations. Students will analyze situations verbally, numerically, graphically, and symbolically. Students will apply mathematical skills and make meaningful connections to life's experiences.

Standard I: Students will expand number sense to understand, perform operations, and solve problems with real numbers.

### Objective 1: Represent real numbers as points on the number line and distinguish rational numbers from irrational numbers.

- a. Define a rational number as a point on the number line that can be expressed as the ratio of two integers, and points that cannot be so expressed as irrational.
- b. Classify numbers as rational or irrational, knowing that rational numbers can be expressed as terminating or repeating decimals and irrational numbers can be expressed as non-terminating, non-repeating decimals.
- c. Classify *pi* and square roots of non-perfect square numbers as irrational.
- d. Place rational and irrational numbers on a number line between two integers.

### Objective 2: Compute fluently and make reasonable estimates with rational and irrational numbers.

- a. Simplify, add, subtract, multiply, and divide expressions with square roots.
- b. Evaluate and simplify numerical expressions containing rational numbers and square roots using the order of operations.
- c. Compute solutions to problems, represent answers in exact form, and determine the reasonableness of answers.
- d. Calculate the measures of the sides of a right triangle using the Pythagorean Theorem.

square root,  $\sqrt{\ }$ , radical, rational, irrational, Pythagorean Theorem

### Standard II: Students will extend concepts of proportion to represent and analyze linear relations.

#### **Objective 1: Represent and analyze the slope of a line.**

- a. Identify the slope of a line when given points, a graph, or an equation.
- b. Identify horizontal and vertical lines given the equations or slopes.
- c. Determine the effect of changes in slope or y-intercept in y = mx + b.
- d. Determine and explain the meaning of slopes and intercepts using real-world examples.

### Objective 2: Model and interpret problems having a constant rate of change using linear functions.

- a. Write algebraic expressions or equations to generalize visual patterns, numerical patterns, relations, data sets, or scatter plots.
- b. Represent linear equations in slope-intercept form, y = mx + b, and standard form, Ax + By = C.
- c. Distinguish between linear and non-linear functions by examining a table, equation, or graph.
- d. Interpret the slope of a linear function as a rate of change in real-world situations.

### Objective 3: Represent and analyze linear relationships using algebraic equations, expressions, and graphs.

- a. Write the equation of a line when given two points or the slope and a point on the line.
- b. Approximate the equation of a line given the graph of a line.
- c. Identify the *x* and *y*-intercepts from an equation or graph of a line or a table of values.
- d. Graph linear relations and inequalities by plotting points, by finding *x* and *y*-intercepts, or by using the slope and any point on the line.

#### Mathematical Language and Symbols Students Should Use

Slope, x-intercept, y-intercept, y = mx + b, Ax + By = C, undefined slope

### Standard III: Students will develop fluency with the language and operations of algebra to analyze and represent relationships.

#### Objective 1: Simplify polynomials and the quotient of monomials.

- a. Simplify and evaluate monomial expressions and formulas.
- b. Add and subtract polynomials.
- c. Multiply monomials by a polynomial.
- d. Multiply binomials.
- e. Simplify the quotient of monomials using positive exponents.

## Objective 2: Solve and interpret linear equations and inequalities in various situations including real-world problems.

- a. Solve single-variable linear equations and inequalities algebraically and graphically.
- b. Solve real-world problems involving constant rates of change.
- c. Solve equations for a specified variable.
- d. Solve proportions that include algebraic first-degree expressions.

#### Objective 3: Solve and interpret pairs of linear equations and inequalities.

- a. Solve systems of two linear equations graphically and algebraically with and without technology.
- b. Determine the number of possible solutions for a system of two linear equations.
- c. Graph a system of linear inequalities and identify the solution.

## Objective 4: Factor polynomials with common monomial factors and factor simple quadratic expressions.

- a. Find the greatest common monomial factor of a polynomial.
- b. Factor trinomials with integer coefficients of the form  $x^2 + bx + c$ .
- c. Factor the difference of two squares and perfect square trinomials.

#### Objective 5: Solve quadratic equations using factoring or by taking square roots.

- a. Solve quadratic equations that can be simplified to the form  $x^2 = a$  where  $a \ge 0$  by taking square roots.
- b. Solve quadratic equations using factoring.
- c. Write a quadratic equation when given the solutions.

#### Mathematical Language and Symbols Students Should Use

monomial, binomial, trinomial, polynomial, literal equation, factor, difference of two squares, perfect square, quadratic

### Standard IV: Students will understand concepts from statistics and apply statistical methods to solve problems.

#### Objective 1: Summarize, display, and analyze bivariate data.

- a. Collect, record, organize, and display a set of data with at least two variables.
- b. Determine whether the relationship between two variables is approximately linear or non-linear by examination of a scatter plot.
- c. Characterize the relationship between two linear related variables as having positive, negative, or approximately zero correlation.

#### Objective 2: Estimate, interpret, and use lines fit to bivariate data.

- a. Estimate the equation of a line of best fit to make and test conjectures.
- b. Interpret the slope and y-intercept of a line through data.
- c. Predict *y*-values for given *x*-values when appropriate using a line fitted to bivariate numerical data.

### **Mathematical Language and Symbols Students Should Use**

scatter plot, positive correlation, negative correlation, no correlation, line of best fit, bivariate

## **Geometry** Prerequisite: Algebra 1

The main goal of Geometry is for students to develop the structure of Euclidean geometry logically and apply the resulting theorems and formulas to address meaningful problems. Students will use experimentation and inductive reasoning to construct geometric concepts, discover geometric relationships, and formulate conjectures. Students will employ deductive logic to construct formal logical arguments and proofs. Students will extend their pre-existing experiences with algebra and geometry to trigonometry and coordinate geometry. Students will use dynamic geometry software, compass and straightedge, and other tools to investigate and explore mathematical ideas and relationships and develop multiple strategies for analyzing complex situations. Students will apply mathematical skills and make meaningful connections to life's experiences.

## Standard I: Students will use algebraic, spatial, and logical reasoning to solve geometry problems.

### Objective 1: Use inductive and deductive reasoning to develop mathematical arguments.

- a. Write conditional statements, converses, and inverses, and determine the truth value of these statements.
- b. Formulate conjectures using inductive reasoning.
- c. Prove a statement false by using a counterexample.

### Objective 2: Analyze characteristics and properties of angles.

- a. Use accepted geometric notation for lines, segments, rays, angles, similarity, and congruence.
- b. Identify and determine relationships in adjacent, complementary, supplementary, or vertical angles and linear pairs.
- c. Classify angle pairs formed by two lines and a transversal.
- d. Prove relationships in angle pairs.
- e. Prove lines parallel or perpendicular using slope or angle relationships.

### Objective 3: Analyze characteristics and properties of triangles.

- a. Prove congruency and similarity of triangles using postulates and theorems.
- b. Prove the Pythagorean Theorem in multiple ways, find missing sides of right triangles using the Pythagorean Theorem, and determine whether a triangle is a right triangle using the converse of the Pythagorean Theorem.
- c. Prove and apply theorems involving isosceles triangles.
- d. Apply triangle inequality theorems.
- e. Identify medians, altitudes, and angle bisectors of a triangle, and the perpendicular bisectors of the sides of a triangle, and justify the concurrency theorems.

#### Objective 4: Analyze characteristics and properties of polygons and circles.

- a. Use examples and counterexamples to classify subsets of quadrilaterals.
- b. Prove properties of quadrilaterals using triangle congruence relationships, postulates, and theorems.
- c. Derive, justify, and use formulas for the number of diagonals, lines of symmetry, angle measures, perimeter, and area of regular polygons.
- d. Define radius, diameter, chord, secant, arc, sector, central angle, inscribed angle, and tangent of a circle, and solve problems using their properties.
- e. Show the relationship between intercepted arcs and inscribed or central angles, and find their measures.

### Objective 5: Perform basic geometric constructions, describing and justifying the procedures used.

- a. Investigate geometric relationships using constructions.
- b. Copy and bisect angles and segments.
- c. Construct perpendicular and parallel lines.
- d. Justify procedures used to construct geometric figures.
- e. Discover and investigate conjectures about geometric properties using constructions.

#### Objective 6: Analyze characteristics and properties of three-dimensional figures.

- a. Identify and classify prisms, pyramids, cylinders and cones based on the shape of their base(s).
- b. Identify three-dimensional objects from different perspectives using nets, crosssections, and two-dimensional views.
- c. Describe the symmetries of three-dimensional figures.
- d. Describe relationships between the faces, edges, and vertices of polyhedra.

### Mathematical Language and Symbols Students Should Use

conditional statement, converse, inverse, conjecture, inductive, deductive, counterexample, adjacent, complementary, supplementary, vertical angles, linear pair, transversal, congruent, postulate, theorem, isosceles, median, altitude, angle bisector, secant, arc, sector, central angle, inscribed angle, tangent of a circle, intercepted arc, construct, bisect, net, polyhedra,  $\overrightarrow{AB}$ ,  $\overrightarrow{$ 

## Standard II: Students will use the language and operations of algebra to explore geometric relationships with coordinate geometry.

### Objective 1: Describe the properties and attributes of lines and line segments using coordinate geometry.

- a. Verify the classifications of geometric figures using coordinate geometry to find lengths and slopes.
- b. Find the distance between two given points and find the coordinates of the midpoint.
- c. Write an equation of a line perpendicular or a line parallel to a line through a given point.

#### Objective 2: Describe spatial relationships using coordinate geometry.

- a. Graph a circle given the equation in the form  $(x-h)^2 + (y-k)^2 = r^2$ , and write the equation when given the graph.
- b. Determine whether points in a set are collinear.

### **Mathematical Language and Symbols Students Should Use**

distance formula, equation of a circle, collinear

### Standard III: Students will extend concepts of proportion and similarity to trigonometric ratios.

#### Objective 1: Use triangle relationships to solve problems.

- a. Solve problems using the properties of special right triangles, e.g.,  $30^{\circ}$ ,  $60^{\circ}$ ,  $90^{\circ}$  or  $45^{\circ}$ ,  $45^{\circ}$ ,  $90^{\circ}$ .
- b. Identify the trigonometric relationships of sine, cosine, and tangent with the appropriate ratio of sides of a right triangle.
- c. Express trigonometric relationships using exact values and approximations.

### Objective 2: Use the trigonometric ratios of sine, cosine, and tangent to represent and solve for missing parts of triangles.

- a. Find the angle measure in degrees when given the trigonometric ratio.
- b. Find the trigonometric ratio given the angle measure in degrees, using a calculator.
- c. Find unknown measures of right triangles using sine, cosine, and tangent functions and inverse trigonometric functions.

### **Mathematical Language and Symbols Students Should Use**

special right triangle, sine (sin), cosine (cos), tangent (tan), exact value

### Standard IV: Students will use algebraic, spatial, and logical reasoning to solve measurement problems.

#### Objective 1: Find measurements of plane and solid figures.

- a. Find linear and angle measures in real-world situations using appropriate tools or technology.
- b. Develop surface area and volume formulas for polyhedra, cones, and cylinders.
- c. Determine perimeter, area, surface area, lateral area, and volume for prisms, cylinders, pyramids, cones, and spheres when given the formulas.
- d. Calculate or estimate the area of an irregular region.
- e. Find the length of an arc and the area of a sector when given the angle measure and radius.

#### Objective 2: Solve real-world problems using visualization and spatial reasoning.

- a. Solve problems using the Pythagorean Theorem and its converse.
- b. Solve problems using the distance formula.
- c. Solve problems involving trigonometric ratios.
- d. Solve problems involving geometric probability.

## Mathematical Language and Symbols Students Should Use polyhedra, cone, cylinder, sphere, arc, area of a sector, geometric probability

## **Algebra 2** Prerequisite: Geometry

A primary goal of Algebra 2 is for students to conceptualize, analyze, and identify relationships among functions. This course builds on concepts learned in Algebra 1 and Geometry by extending linear algebra and coordinate geometry concepts to other functions and systems of equations. Students will develop proficiency in analyzing and solving quadratic functions using complex numbers. Students will investigate and make conjectures about absolute value, radical, exponential, logarithmic and sine and cosine functions algebraically, numerically, and graphically, with and without a graphing calculator. Students will extend their algebraic skills to compute with rational expressions and rational exponents. Students will analyze statistical data and apply concepts of probability using permutations and combinations. Students will apply mathematical skills and make meaningful connections to life's experiences.

## Standard I: Students will use the language and operations of algebra to evaluate, analyze and solve problems.

### Objective 1: Evaluate, analyze, and solve mathematical situations using algebraic properties and symbols.

- a. Solve and graph first-degree absolute value equations of a single variable.
- b. Solve radical equations of a single variable, including those with extraneous roots.
- c. Solve absolute value and compound inequalities of a single variable.
- d. Add, subtract, multiply, and divide rational expressions and solve rational equations.
- e. Simplify algebraic expressions involving negative and rational exponents.

#### Objective 2: Solve systems of equations and inequalities.

- a. Solve systems of linear, absolute value, and quadratic equations algebraically and graphically.
- b. Graph the solutions of systems of linear, absolute value, and quadratic inequalities on the coordinate plane.
- c. Solve application problems involving systems of equations and inequalities.

#### Objective 3: Represent and compute fluently with complex numbers.

- a. Simplify numerical expressions, including those with rational exponents.
- b. Simplify expressions involving complex numbers and express them in standard form, a + bi.

#### Objective 4: Model and solve quadratic equations and inequalities.

- a. Model real-world situations using quadratic equations.
- b. Approximate the real solutions of quadratic equations graphically.
- c. Solve quadratic equations of a single variable over the set of complex numbers by factoring, completing the square, and using the quadratic formula.
- d. Solve quadratic inequalities of a single variable.
- e. Write a quadratic equation when given the solutions of the equation.

#### **Mathematical Language and Symbols Students Should Use**

compound inequality, rational equation, system of equations, complex number, completing the square, quadratic formula

### Standard II: Students will understand and represent functions and analyze function behavior.

#### Objective 1: Represent mathematical situations using relations.

- a. Model real-world relationships with functions.
- b. Describe a pattern using function notation.
- c. Determine when a relation is a function.
- d. Determine the domain and range of relations.

#### Objective 2: Evaluate and analyze functions.

- a. Find the value of a function at a given point.
- b. Compose functions when possible.
- c. Add, subtract, multiply, and divide functions.
- d. Determine whether or not a function has an inverse, and find the inverse when it exists.
- e. Identify the domain and range of a function resulting from the combination or composition of functions.

### Objective 3: Define and graph exponential functions and use them to model problems in mathematical and real-world contexts.

- a. Define exponential functions as functions of the form  $y = ab^x$ , b > 0,  $b \ne 1$ .
- b. Model problems of growth and decay using exponential functions.
- c. Graph exponential functions.

### Objective 4: Define and graph logarithmic functions and use them to solve problems in mathematics and real-world contexts.

- a. Relate logarithmic and exponential functions.
- b. Simplify logarithmic expressions.
- c. Convert logarithms between bases.
- d. Solve exponential and logarithmic equations.
- e. Graph logarithmic functions.
- f. Solve problems involving growth and decay.

#### Mathematical Language and Symbols Students Should Use

function, relation, domain, range, f(x), f(g(x)),  $f \circ g$ , one to one, inverse, exponential function, logarithm, base, e

### Standard III: Students will use algebraic, spatial, and logical reasoning to solve geometry and measurement problems.

#### **Objective 1: Examine the behavior of functions using coordinate geometry.**

- a. Identify the domain and range of the absolute value, quadratic, radical, sine, and cosine functions.
- b. Graph the absolute value, quadratic, radical, sine, and cosine functions.
- c. Graph functions using transformations of parent functions.
- d. Write an equation of a parabola in the form  $y = a(x h)^2 + k$  when given a graph or an equation.

#### Objective 2: Determine radian and degree measures for angles.

- a. Convert angle measurements between radians and degrees.
- b. Find angle measures in degrees and radians using inverse trigonometric functions, including exact values for special triangles.

### Objective 3: Determine trigonometric measurements using appropriate techniques, tools, and formulas.

- a. Define the sine, cosine, and tangent functions using the unit circle.
- b. Determine the exact values of the sine, cosine, and tangent functions for the special angles of the unit circle using reference angles.
- c. Find the length of an arc using radian measure.
- d. Find the area of a sector in a circle using radian measure.

#### **Mathematical Language and Symbols Students Should Use**

transformation, parabola, radian, unit circle, reference angle

### Standard IV: Students will understand concepts from probability and statistics and apply statistical methods to solve problems.

#### Objective 1: Apply basic concepts of probability.

- a. Distinguish between permutations and combinations and identify situations in which each is appropriate.
- b. Calculate probabilities using permutations and combinations to count events.
- c. Compute conditional and unconditional probabilities in various ways, including by definitions, the general multiplication rule, and probability trees.
- d. Define simple discrete random variables.

#### Objective 2: Use percentiles and measures of variability to analyze data.

- a. Compute different measures of spread, including the range, standard deviation, and interquartile range.
- b. Compare the effectiveness of different measures of spread, including the range, standard deviation, and interquartile range in specific situations.
- c. Use percentiles to summarize the distribution of a numerical variable.
- d. Use histograms to obtain percentiles.

### **Mathematical Language and Symbols Students Should Use**

permutation, combination, conditional probability, discrete random variable, standard deviation, interquartile range, percentile

## **Precalculus Prerequisite: Algebra 2**

The main goal of Precalculus is for students to gain a deep understanding of the fundamental concepts and relationships of functions. Students will expand their knowledge of quadratic, exponential, and logarithmic functions to include power, polynomial, rational, piece-wise, and trigonometric functions. Students will investigate and explore mathematical ideas, develop multiple strategies for analyzing complex situations, and use graphing calculators and mathematical software to build understanding, make connections between representations, and provide support in solving problems. Students will analyze various representations of functions, sequences, and series. Students will analyze bivariate data and data distributions. Students will apply mathematical skills and make meaningful connections to life's experiences. Precalculus is highly recommended preparation for students who plan to continue their formal education beyond high school.

## Standard I: Students will use the language and operations of algebra to evaluate, analyze and solve problems.

#### Objective 1: Compute with matrices and use matrices to solve problems.

- a. Represent real-world situations with matrices.
- b. Add, subtract, and multiply (including scalar multiplication) matrices using paper and pencil, and computer programs or calculators.
- c. Demonstrate that matrix multiplication is associative and distributive, but not commutative.
- d. Determine additive and multiplicative identities and inverses of a matrix when they exist.
- e. Solve systems of linear equations with up to three variables using matrices.

#### Objective 2: Analyze the behavior of sequences and series.

- a. Describe a sequence as a function where the domain is the set of natural numbers.
- b. Represent sequences and series using various notations.
- c. Identify arithmetic and geometric sequences and series.
- d. Discover and justify the formula for a finite arithmetic series.
- e. Discover and justify the formulas for finite and infinite geometric series.

# Mathematical Language and Symbols Students Should Use matrix, scalar, sequence, series, arithmetic sequence, arithmetic series, geometric sequence, geometric series, $\sum$

### Standard II: Students will understand and represent functions and analyze function behavior.

#### Objective 1: Analyze and solve problems using polynomial functions.

- a. Raise a binomial to a power using the Binomial Theorem and Pascal's Triangle.
- b. Determine the number and nature of solutions to polynomial equations with real coefficients over the complex numbers.
- c. Factor polynomials to solve equations and real-world applications.
- d. Understand the relationships among the solutions of a polynomial equation, the zeros of a function, the *x*-intercepts of a graph, and the factors of a polynomial.
- e. Write an equation with given solutions.

### Objective 2: Model and graph functions and transformations of functions.

- a. Model real-world relationships with functions.
- b. Graph rational, piece-wise, power, exponential, and logarithmic functions.
- c. Identify the effects of changing the parameter a in y = af(x), y = f(ax), y = f(x-a), and y = f(x) + a, given the graph of y = f(x).

#### Objective 3: Analyze the behavior of functions.

- a. Identify the domain, range, and other attributes of families of functions and their inverses.
- b. Approximate instantaneous rates of change and find average rates of change using graphs and numerical data.
- c. Identify and analyze continuity, end behavior, asymptotes, symmetry (odd and even functions), and limits, and connect these concepts to graphs of functions.
- d. Determine intervals over which a function is increasing or decreasing, and describe the intervals using interval notation.
- e. Relate the graphical representation of discontinuities and end behavior to the concept of limit.

#### Mathematical Language and Symbols Students Should Use

Binomial Theorem, rate of change, interval, asymptote, hole, extrema, discontinuous, continuous, odd and even function, limit,  $\sum$ 

## Standard III: Students will use algebraic, spatial, and logical reasoning to solve geometry and measurement problems.

#### **Objective 1: Solve problems using trigonometry.**

- a. Define the six trigonometric functions using the unit circle.
- b. Prove trigonometric identities using definitions, the Pythagorean Theorem, or other relationships.
- c. Simplify trigonometric expressions and solve trigonometric equations using identities.
- d. Solve problems using the Law of Sines and the Law of Cosines.
- e. Construct the graphs of the trigonometric functions and their inverses, and describe their behavior, including periodicity and amplitude.

#### Objective 2: Graph curves using polar and parametric equations.

- a. Define and use polar coordinates and relate them to Cartesian coordinates.
- b. Represent complex numbers in rectangular and polar form, and convert between rectangular and polar form.
- c. Translate equations in Cartesian coordinates into polar coordinates and graph them in the polar coordinate plane.
- d. Multiply complex numbers in polar form and use DeMoivre's Theorem to find roots of complex numbers.
- e. Define a curve parametrically and draw parametric graphs.

### Objective 3: Solve problems involving the geometric properties of conic sections.

- a. Write equations of conic sections in standard form.
- b. Identify the geometric properties of conic sections (i.e., vertex, foci, lines of symmetry, directrix, major and minor axes, and asymptotes).
- c. Solve real-world applications of conic sections.

#### Mathematical Language and Symbols Students Should Use

Law of Sines, Law of Cosines, conic section, ellipse, hyperbola, secant (sec), cosecant (csc), cotangent (cot), polar coordinates, parametric

### Standard IV: Students will understand concepts from probability and statistics and apply statistical methods to solve problems.

### Objective 1: Compute probabilities for discrete distributions and use sampling distributions to calculate approximate probabilities.

- a. Obtain sample spaces and probability distributions for simple discrete random variables.
- b. Compute binomial probabilities using Pascal's Triangle and the Binomial Theorem.
- c. Compute means and variances of discrete random variables.
- d. Compute probabilities using areas under the Normal Curve.
- e. Calculate parameters of sampling distributions for the sample average, sum, and proportion.
- f. Calculate probabilities in real problems using sampling distributions.

#### Objective 2: Analyze bivariate data using linear regression methods.

- Fit regression lines to pairs of numeric variables and calculate the means and standard deviations of the two variables and the correlation coefficient, using technology.
- b. Compute predictions of *y*-values for given *x*-values using a regression equation, and recognize the limitations of such predictions.
- c. Compute and use the standard error for regression.

### Mathematical Language and Symbols Students Should Use

regression line, correlation coefficient, standard error